

**AFRL-ML-WP-TP-2006-501**

**A NEW APPROACH TO FATIGUE  
LIFE PREDICTION BASED ON  
NUCLEATION AND GROWTH  
(PREPRINT)**

**R.C. McClung, W.L. Francis, and S.J. Hudak, Jr.**



**FEBRUARY 2006**

**Approved for public release; distribution is unlimited.**

**STINFO COPY**

**This work was funded in whole or in part by Department of the Air Force contract F33615-03-2-5203. The U.S. Government has for itself and others acting on its behalf an unlimited, paid-up, nonexclusive, irrevocable worldwide license to use, modify, reproduce, release, perform, display, or disclose the work by or on behalf of the U.S. Government.**

**MATERIALS AND MANUFACTURING DIRECTORATE  
AIR FORCE RESEARCH LABORATORY  
AIR FORCE MATERIEL COMMAND  
WRIGHT-PATTERSON AIR FORCE BASE, OH 45433-7750**

<b>REPORT DOCUMENTATION PAGE</b>				<i>Form Approved</i> OMB No. 0704-0188	
The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. <b>PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.</b>					
<b>1. REPORT DATE (DD-MM-YY)</b> February 2006		<b>2. REPORT TYPE</b> Conference Paper Preprint		<b>3. DATES COVERED (From - To)</b>	
<b>4. TITLE AND SUBTITLE</b> A NEW APPROACH TO FATIGUE LIFE PREDICTION BASED ON NUCLEATION AND GROWTH (PREPRINT)				<b>5a. CONTRACT NUMBER</b> F33615-03-2-5203	
				<b>5b. GRANT NUMBER</b>	
				<b>5c. PROGRAM ELEMENT NUMBER</b> 62102F	
<b>6. AUTHOR(S)</b> R.C. McClung, W.L. Francis, and S.J. Hudak, Jr.				<b>5d. PROJECT NUMBER</b> 4347	
				<b>5e. TASK NUMBER</b> 27	
				<b>5f. WORK UNIT NUMBER</b> 02	
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b>  Southwest Research Institute San Antonio, TX 78228-0510				<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>	
<b>9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b> Materials and Manufacturing Directorate Air Force Research Laboratory Air Force Materiel Command Wright-Patterson AFB, OH 45433-7750				<b>10. SPONSORING/MONITORING AGENCY ACRONYM(S)</b> AFRL-ML-WP	
				<b>11. SPONSORING/MONITORING AGENCY REPORT NUMBER(S)</b> AFRL-ML-WP-TP-2006-501	
<b>12. DISTRIBUTION/AVAILABILITY STATEMENT</b> Approved for public release; distribution is unlimited.					
<b>13. SUPPLEMENTARY NOTES</b> Conference paper submitted to the Proceedings of 9th International Fatigue Congress (Fatigue 2006), published by Elsevier Ltd. This work was funded in whole or in part by Department of the Air Force contract F33615-03-2-5203. The U.S. Government has for itself and others acting on its behalf an unlimited, paid-up, nonexclusive, irrevocable worldwide license to use, modify, reproduce, release, perform, display, or disclose the work by or on behalf of the U.S. Government. Paper contains color. PAO Case Number: AFRL/WS 06-0305; Date cleared: 06 Feb 2006.					
<b>14. ABSTRACT</b> Prediction of total fatigue life in components is often performed by summing "initiation" and "propagation" life phases, where initiation life is based on stress-life or strain-life methods calibrated to smooth specimen fatigue tests. An "engineering" size crack (e.g., 0.030" or 1 mm) is often used as the transition between initiation and propagation analyses. However, these methods commonly fail to give accurate predictions for problems with significant stress gradients. A new approach to fatigue life prediction has been developed to address this shortcoming. The approach employs traditional smooth specimen and fatigue crack growth data but applies them in novel ways. The first step is to generate "nucleation" curves to a designated small crack size from smooth specimen data by subtracting calculated crack growth life (including small-crack corrections) in the smooth specimen from the total smooth specimen life. The second step is to apply these nucleation curves to predict the nucleation life to the same initial crack size in a feature of interest, equating the stress at the nucleation length in the feature geometry with the uniform stress in a corresponding virtual smooth specimen. The feature crack growth life is calculated from this crack size to failure, and then nucleation and growth lives are summed to obtain the total fatigue life. The approach has been demonstrated for Ti-6Al-4V using available data from smooth specimen and fatigue crack growth tests to predict total fatigue lives in double-edge notched fatigue specimens at three different stress ratios.					
<b>15. SUBJECT TERMS</b> Fatigue life prediction, small cracks, stress gradients, crack nucleation, crack growth					
<b>16. SECURITY CLASSIFICATION OF:</b>			<b>17. LIMITATION OF ABSTRACT:</b> SAR	<b>18. NUMBER OF PAGES</b> 16	<b>19a. NAME OF RESPONSIBLE PERSON (Monitor)</b> Patrick Golden <b>19b. TELEPHONE NUMBER (Include Area Code)</b> N/A
<b>a. REPORT</b> Unclassified	<b>b. ABSTRACT</b> Unclassified	<b>c. THIS PAGE</b> Unclassified			

# **A NEW APPROACH TO FATIGUE LIFE PREDICTION BASED ON NUCLEATION AND GROWTH**

R. C. McClung, W. L. Francis, and S. J. Hudak, Jr.

Southwest Research Institute  
San Antonio, TX 78228-0510, USA

## **ABSTRACT**

Prediction of total fatigue life in components is often performed by summing “initiation” and “propagation” life phases, where initiation life is based on stress-life or strain-life methods calibrated to smooth specimen fatigue tests. An “engineering” size crack (e.g., 0.030” or 1 mm) is often used as the transition between initiation and propagation analyses. However, these methods commonly fail to give accurate predictions for problems with significant stress gradients. A new approach to fatigue life prediction has been developed to address this shortcoming. The approach employs traditional smooth specimen and fatigue crack growth data but applies them in novel ways. The first step is to generate “nucleation” curves to a designated small crack size from smooth specimen data by subtracting calculated crack growth life (including small-crack corrections) in the smooth specimen from the total smooth specimen life. The second step is to apply these nucleation curves to predict the nucleation life to the same initial crack size in a feature of interest, equating the stress at the nucleation length in the feature geometry with the uniform stress in a corresponding virtual smooth specimen. The feature crack growth life is calculated from this crack size to failure, and then nucleation and growth lives are summed to obtain the total fatigue life. The approach has been demonstrated for Ti-6Al-4V using available data from smooth specimen and fatigue crack growth tests to predict total fatigue lives in double-edge notched fatigue specimens at three different stress ratios.

## **KEYWORDS**

Fatigue life prediction, small cracks, stress gradients, crack nucleation, crack growth

## **INTRODUCTION**

Prediction of fatigue life at stress concentrations is a classical problem. Traditional stress-life approaches in which smooth specimen behavior is modified by the theoretical stress concentration factor ( $k_t$ ) give overly conservative results. Semi-empirical corrections to this approach employing a diminished “fatigue notch factor” ( $k_f$ ) give improved results but do not account for some stress level and geometry effects.

Better methods directly address the partitioning of life into crack “initiation” and “propagation” phases, independently calculating each phase with appropriate methods and then summing the two to obtain total life. Socie et al. [1] and Dowling [2] are representative notable early treatments of this type; space does not permit a more detailed discussion of the literature. The transition length at which initiation and propagation methods are joined in these methods can vary significantly with notch geometry and stress level.

It is common in some industrial practice today to associate crack “initiation life” with the formation of a crack with some significant (fixed) engineering size (e.g., 0.030 in. or 1 mm). This “initiation life” is often related to the fatigue failure of a smooth cylindrical specimen under uniform axial loading. A corresponding crack “propagation life” is then calculated using the same length as the initial flaw size. This scheme can also give over-conservative life results, especially for small stress concentrations and steep stress gradients. This method neglects two important phenomena: much of the “initiation life” is actually consumed by the growth of a very small crack, and the stresses may decrease substantially over the initiation length.

In this work, a new approach is outlined and demonstrated for fatigue life prediction in significant stress gradients. The approach is based on the simple summation of crack nucleation and growth life phases, but with the transition occurring at a physically meaningful crack nucleation size. The approach employs traditional stress-life (smooth specimen) and large-crack fracture mechanics properties while also addressing important small-crack and stress gradient phenomena.

## APPROACH

A smooth specimen fatigue test actually includes a crack growth test, because a portion of the total life is spent growing a nucleated crack to failure. It is possible to estimate the number of cycles required to nucleate a “little” crack of some specified size in a smooth specimen by calculating the number of cycles required to grow this initial crack to failure in a smooth specimen, and then subtracting the calculated crack growth life from the observed total fatigue life. This process is summarized schematically in Figure 1. A family of “crack nucleation” curves corresponding to different nucleation lengths can be derived from a single smooth specimen total life curve.

This crack growth computation can employ traditional large-crack fracture mechanics methods and properties as long as the crack is adequately large compared to the material microstructure and any local plastic zone sizes associated with either the stress concentration or the crack itself. When the crack is smaller, alternative analysis methods may be required. A simple engineering approach employed here is based on the suggestion of El Haddad [3] that the effective driving force for a small crack is

$$\Delta K_{eq} = F(a)\Delta S\sqrt{\pi(a+a_0)} \quad (1)$$

where  $a_0$  is defined by the intersection of the threshold and endurance limit lines in the Kitagawa [4] diagram construction according to

$$a_0 = \frac{1}{\pi} \left( \frac{\Delta K_{th}}{F\Delta\sigma_e} \right)^2 \quad (2)$$

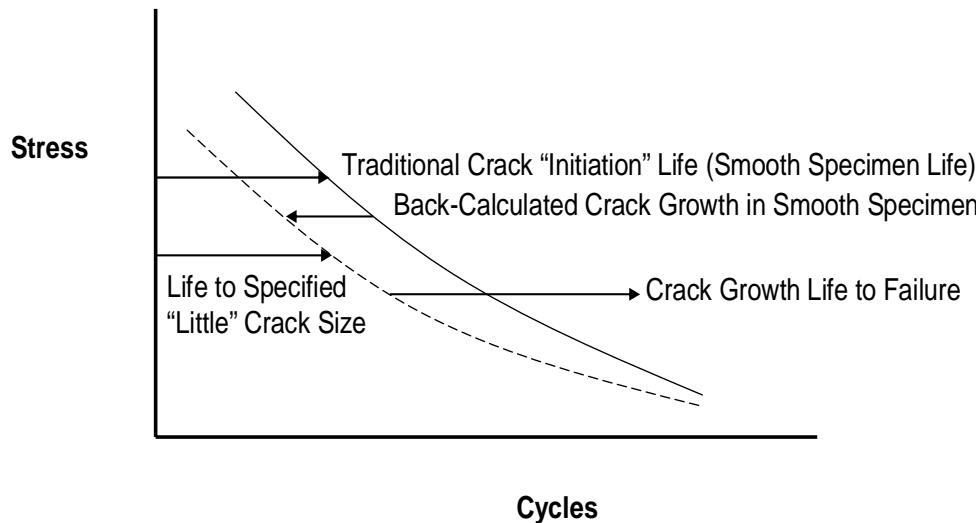


Figure 1. Schematic representation of method to generate nucleation life curves.

Here  $\Delta S$  is a representation of the nominal applied stress,  $a$  is the physical crack length, and  $F(a)$  is the correction factor for the geometry of interest. Although the El Haddad concept has been criticized for its apparent lack of physical justification, several researchers [5, 6] have independently derived similar formulations based on detailed micromechanical considerations, and these have been shown to be numerically identical to the El Haddad formulation under typical conditions. Chan [5] related  $a_0$  to dislocation pileups and mode II shear cracks, and found satisfactory agreement with small-crack data from titanium aluminides. Tanaka [6] formulated the problem in terms of a crack-tip slip band blocked by a grain boundary, and demonstrated reasonable agreement between his formulation and a variety of experimental data. While titanium was not included in the Tanaka comparisons, Brown and Taylor [7] have shown good agreement with the El Haddad formulation for a mill-annealed Ti-6Al-4V.

The crack nucleation calculation for an application problem with a significant stress gradient (e.g., a sharp notch) is performed by determining the local stresses at the nucleation length in the corresponding (uncracked) application geometry, and then determining the nucleation life (at this nucleation length) for a smooth specimen subjected to the same local stresses. The local stress calculation in the application geometry should address any elastic-plastic behavior that may occur if local stresses exceed yield at the stress concentration.

The crack growth life in the application geometry is performed using customary fracture mechanics analysis methods, employing stress intensity factor solutions that address the local geometry and stress gradients in the application problem. The same small-crack correction used to derive the nucleation curves must be used here to compute crack growth life. However, it should be noted that the small-crack correction should not be used at crack sizes so small that severe microstructural influences come into play; a traditional rule-of-thumb is at least two times the grain size, and preferably about five times the grain size. This serves to define a practical lower limit on the nucleation length. In principle, the nucleation length could be decreased further if suitable fracture mechanics methods were available to characterize the growth of microstructurally-small fatigue cracks. On the other hand, it is not clear that macroscopic calculations of the local stresses (employed in the nucleation calculation) would be meaningful at this microscopic level, and so it may be prudent to maintain the same practical lower limit.

Finally, the total fatigue life for the application geometry is determined by summing nucleation and growth contributions. Because both life phases are physically-meaningful values, it is also possible to make a realistic assessment of the relative importance of nucleation and growth for total life.

## DEMONSTRATION

This new approach is demonstrated here for a forged Ti-6Al-4V titanium alloy representative of a turbine engine fan blade material. The forgings were solution treated at 932°C (1710°F) and then fan cooled and vacuum annealed at 704°C (1300°F) for 2 hours. The resulting microstructure contained approximately 60% primary alpha and transformed beta lamellas. The yield and ultimate tensile strengths averaged 930 MPa (134.9 ksi) and 978 MPa (141.8 ksi), respectively. The same material source was employed extensively for high cycle fatigue research in a large U.S. Air Force program [8], and in fact all of the test data presented here were generated by various participants in that program.

Smooth specimen fatigue data [9] (6.35 mm (0.25 in.) diameter axial specimens) for three different stress ratios are summarized in Figure 2. Also shown in Figure 2 is a convenient regression of the total life data based on a modified Smith-Watson-Topper (SWT) equation,

$$\sqrt{\sigma_{max} \epsilon_{a,max}} E = 59263 N^{-0.59} + 517 N^{-0.023} \quad (3)$$

where  $\epsilon_{a,max}$  is the maximum principal strain amplitude,  $\sigma_{max}$  is the maximum normal stress on the critical plane of  $\epsilon_{a,max}$ ,  $E$  is Young's modulus, and  $N$  is total cycles to failure.

Large crack fracture mechanics data at various stress ratios were collected from multiple sources [10, 11] along with limited small-crack data [12], all for the same lot of material. The large-crack and small-crack data at  $R = 0.1$  are shown in Figure 3 (top), indicating the accelerated growth rates of the small cracks. The ability of the El Haddad correction factor to correlate the small-crack data with the large-crack trends is shown in Figure 3 (bottom). Here  $a_0$  was calculated to be 0.002” (51  $\mu\text{m}$ ).

Smooth specimen crack nucleation curves were generated from the smooth specimen total life data and the fracture mechanics data according to the construction of Figure 1, and the results at  $R = 0.1$  are shown in Figure 4 for multiple values of the crack nucleation length. Note that a nucleation length of 0.001” (25  $\mu\text{m}$ ) is slightly less than two times the average grain size, while a nucleation length of 0.003” (76  $\mu\text{m}$ ) is about five times the average grain size.

Evaluation of the model requires comparison with fatigue data from a feature geometry involving a significant stress gradient. A series of fatigue tests performed on double edge notch (DEN) specimens [8] was used for this purpose. The DEN specimens contained 90° V-shaped notches with a depth of 0.047” (1.19 mm) and a root radius of 0.021” (0.53 mm) in a cross-section of thickness 0.142” (3.60 mm) and original width (prior to machining the edge notches) of 0.236” (6.0 mm).

Three-dimensional elastic finite element analysis was performed to obtain the stress gradients ahead of the notch root. The nominal (net-section) stress concentration factor was approximately 2.5. The maximum elastic stress decreased from its maximum value at the notch root by a factor of two within 0.013” (0.33 mm) of the notch root.

If the elastic maximum stress in the notched specimen was calculated to be greater than yield, then a shakedown algorithm was invoked to calculate the elastic-plastic stress relaxation and redistribution near the notch root. When shakedown occurred, the local stress ratio was (in general) different from the nominal (applied) stress ratio, and the actual local stress ratio was considered in both the crack nucleation and growth life calculations. The cyclic stress-strain properties of Ti-6Al-4V were employed in the shakedown calculations; the cyclic yield strength is about 780 MPa. An example of the original elastic and shakedown elastic-plastic stress gradients is provided in Figure 5 for nominal  $R = 0.5$  loading with a maximum nominal applied stress of 396 MPa (57.5 ksi). This figure also illustrates the definition of the “nucleation stress” at the specified nucleation length (in this case, 0.003” or 76  $\mu\text{m}$ ).

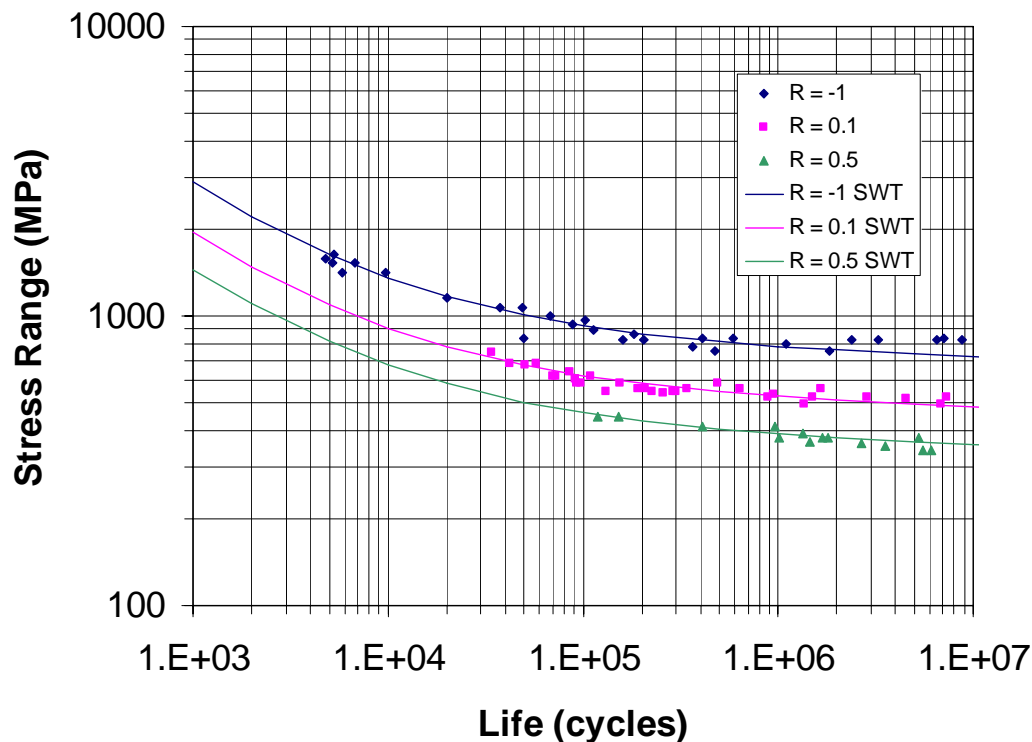


Figure 2. Ti-6Al-4V smooth specimen stress-life data and Smith-Watson-Topper regressions.

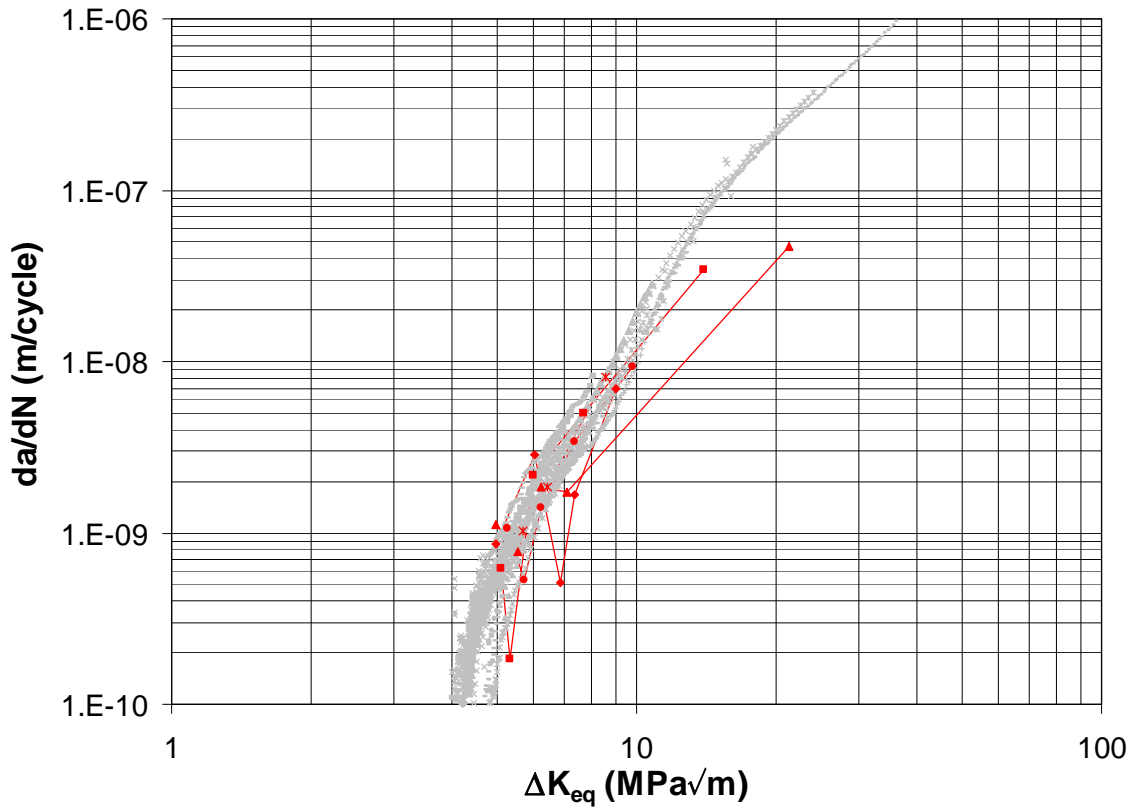
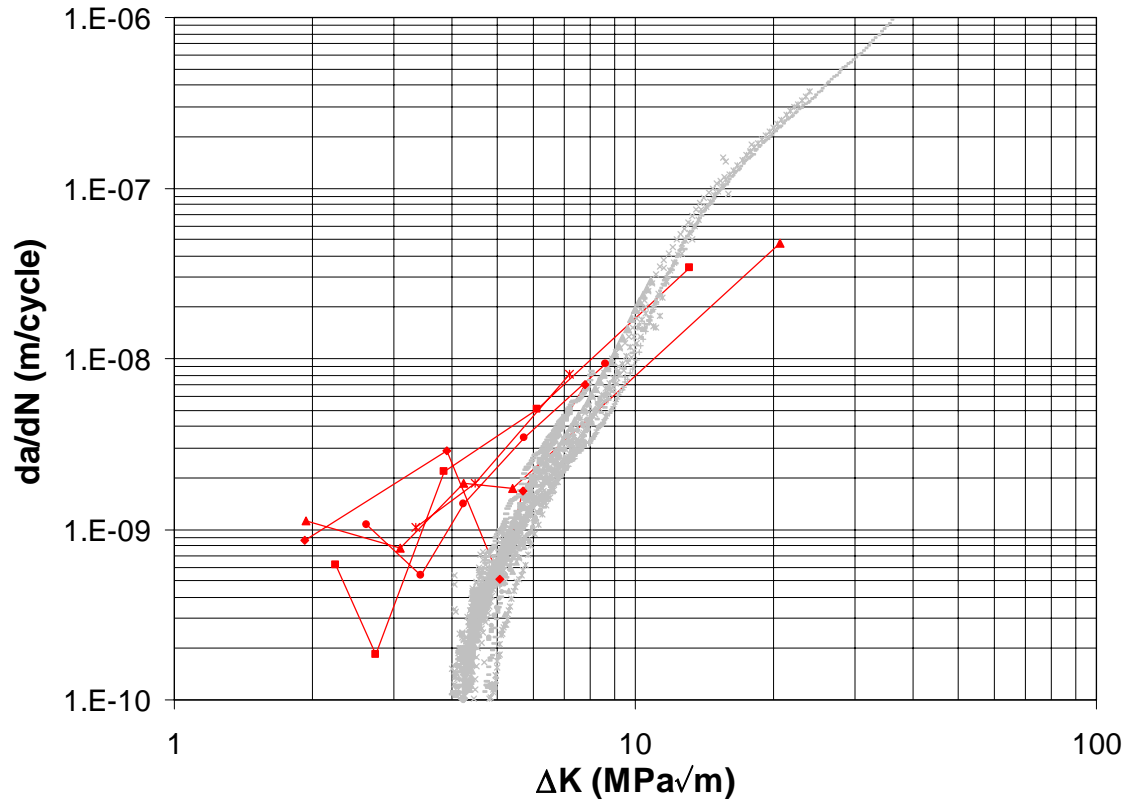


Figure 3. Comparison of small-crack (red symbols) and large-crack (grey symbols) FCG data for Ti-6Al-4V at  $R = 0.1$ . Top graph compares data on basis of nominal  $\Delta K$  while bottom graph shows small-crack data with El Haddad correction.

The test reports indicated that most of the observed cracking initiated as a dominant semi-elliptical surface crack near the center of the notch root, so the initial conditions for the crack growth analysis were selected as a semi-circular crack (with depth equal to the nucleation length) at mid-section.

Several different values of the nucleation length were considered during early investigations. Note that as the nucleation length is decreased, the nucleation life associated with that length decreases, but the crack growth life to failure associated with that nucleation length increases. The calculated total life (nucleation + growth) will not change with nucleation length in the absence of a stress gradient. When the local nucleation stress changes significantly with nucleation length, however, the calculated total life will change with nucleation length.

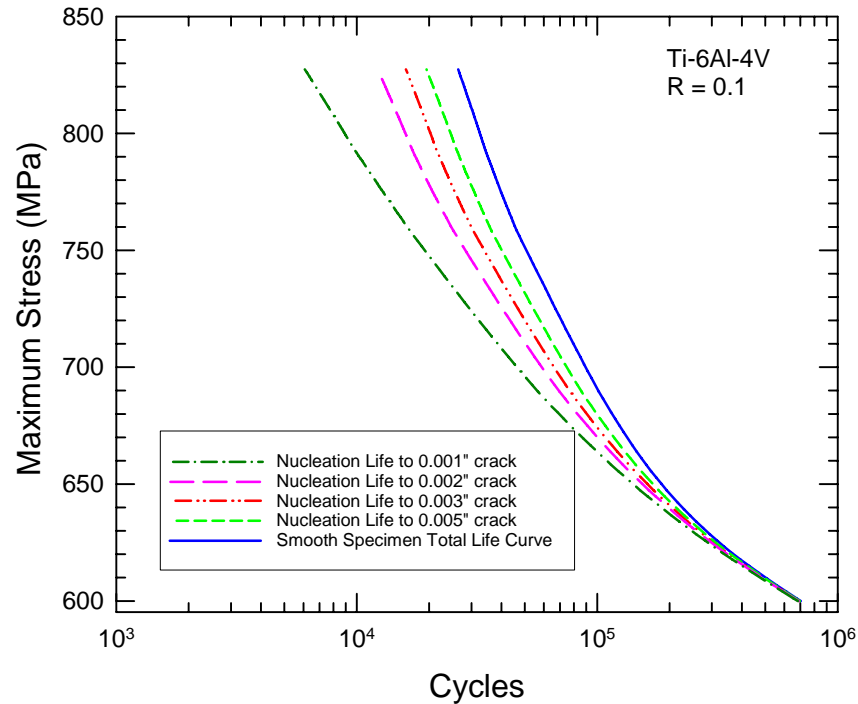


Figure 4. Calculated crack nucleation curves for Ti-6Al-4V at  $R = 0.1$  for different nucleation lengths.

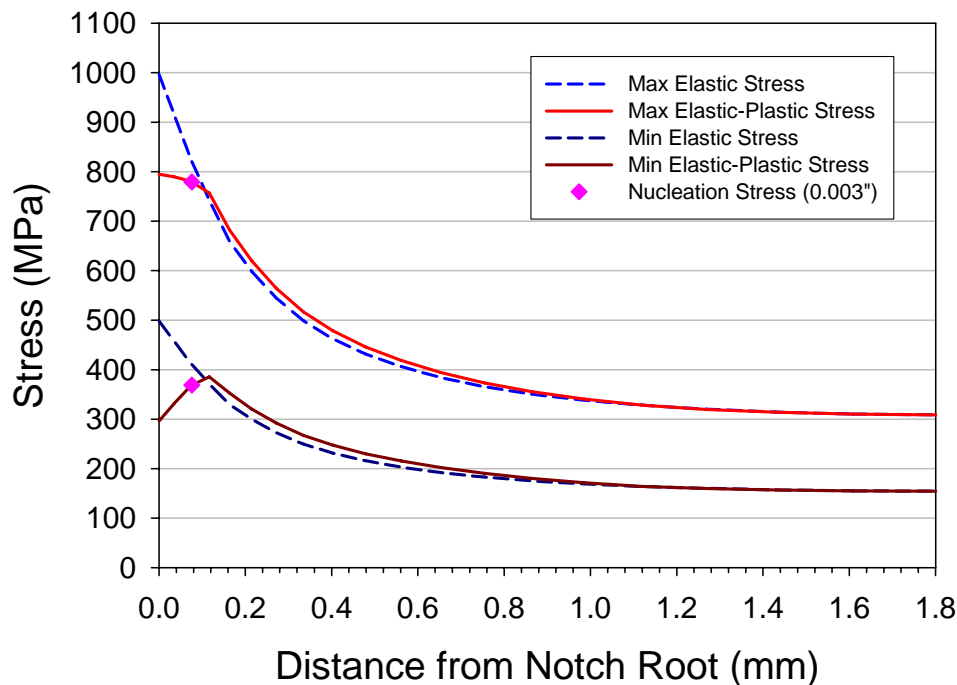


Figure 5. Elastic and elastic-plastic stress gradients at maximum and minimum load for DEN specimen under nominal  $R = 0.5$  loading with a maximum nominal applied stress of 396 MPa (57.5 ksi).



Comparisons of model predictions for total life with DEN test data at  $R = 0.1$  for three different values of the nucleation length are shown in Figure 6. The three different values of nucleation length give similar total life predictions at higher maximum stresses, because yielding near the notch root (which causes stress relaxation and redistribution) substantially reduces the severity of the stress gradient. At lower maximum stresses, different nucleation lengths have a moderate effect on the predicted total life. A nucleation length of about 0.003" (76  $\mu\text{m}$ ) generally gave the most accurate predictions.

Also shown in Figure 6 for reference purposes are total life predictions based on a conventional stress-life approach in which the modified Smith-Watson-Topper model is applied to the elastically-calculated stresses and strains at the notch root itself (essentially, the SWT model is scaled by the peak stress concentration factor). As noted in the Introduction, this type of simple approach substantially overestimates the fatigue damage caused by the notch and gives over-conservative total life estimates. Plastic corrections to the local stresses and strains (non-negligible only above 315 MPa) would be inadequate to reconcile the differences.

Similar results were found for comparisons between predictive models and DEN fatigue test data at stress ratios of  $R = 0.5$  and  $-1$ . In all cases, a nucleation length of 0.003" (76  $\mu\text{m}$ ) gave excellent total life predictions, and SWT predictions scaled by the elastic stress concentration factor gave severely conservative life predictions. The predictions based on a 0.003" nucleation length are summarized for all three stress ratios in Figure 7. It should be emphasized that these predictions are in no way a "fit" to the notch test data (other than the selection of a common nucleation length) because the material constants in the model were derived only from independent smooth specimen and fracture mechanics tests.

The model facilitates evaluation of the fraction of total fatigue life consumed by crack nucleation, for the specified nucleation length. These predictions are summarized in Figure 8 for the same three stress ratios and the same nucleation length (0.003"). In the long life regime, the total life is dominated by crack nucleation, but in the low cycle regime (total lifetimes of  $10^5$  cycles and less), the crack growth phase consumes 30% and more of the total life. It is interesting to note that, for this particular geometry, the crack nucleation life fraction for a given total life does not change significantly with stress ratio.

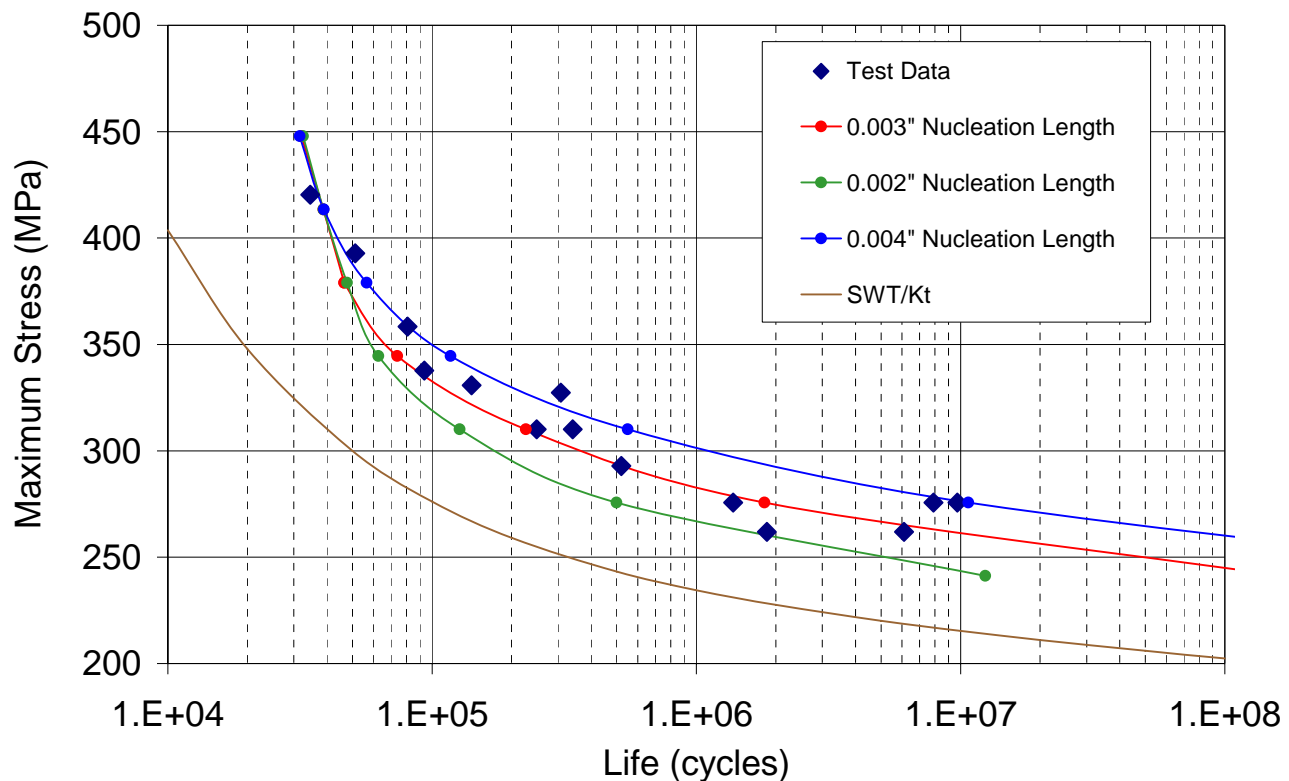


Figure 6. Comparison of  $R = 0.1$  DEN test data with model predictions.

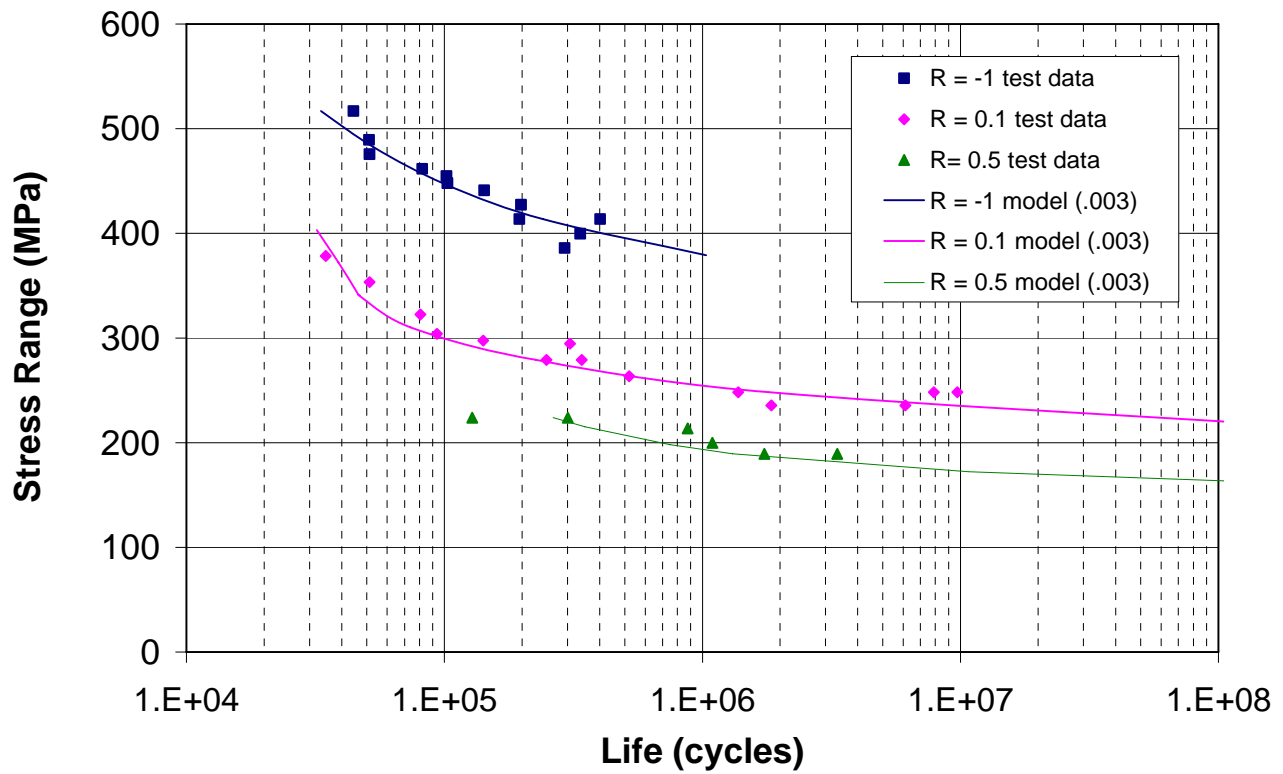


Figure 7. Comparison of DEN test data at three stress ratios with model predictions.

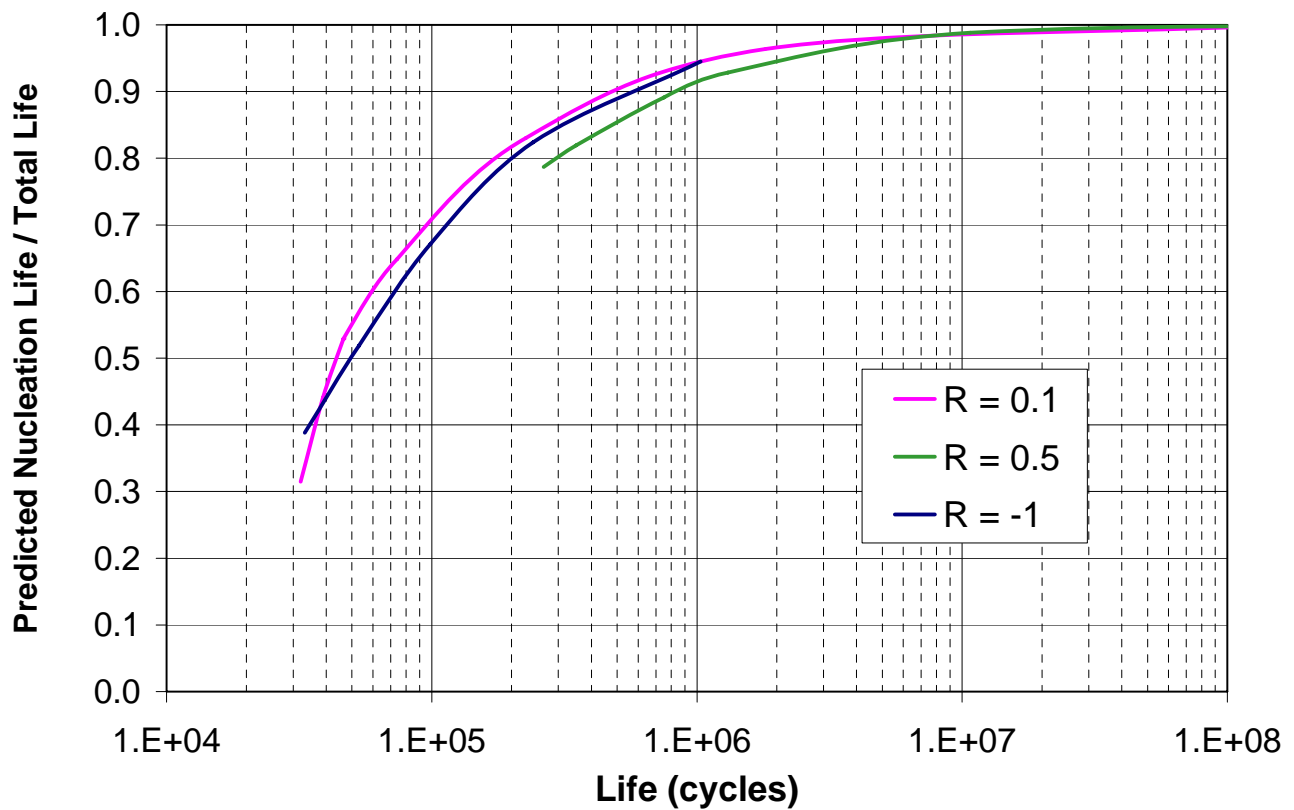


Figure 8. Predicted nucleation life fraction for DEN test specimen at different applied stress ratios.

## DISCUSSION

This study must be regarded as a preliminary investigation due to its limited scope. Further work is needed to evaluate the method for other materials and other feature geometries. Selection of the optimum “nucleation length” also requires further study. It is not yet clear if this length is dependent on material parameters, geometry parameters, stress gradient parameters, or some combination of the three. Although the nucleation length does not always appear to play a significant role in the quality of the life prediction, it is not yet possible to make an *a priori* determination of the only free parameter in the analysis scheme.

Nevertheless, the results of this first study are encouraging. In principle, the method should apply not only to stress concentrations, but also to other problems with significant stress gradients. For example, many surface enhancement methods (such as peening) induce a severe stress gradient very close to the surface. The new method provides a rational basis to partition the problem into physically-meaningful nucleation and growth phases, to assess the relative importance of each, and to obtain improved life predictions.

## SUMMARY AND CONCLUSIONS

A new approach to fatigue life prediction has been developed to address fatigue failure in significant stress gradients. The first step is to generate “nucleation” curves to a designated small crack size from smooth specimen data by subtracting calculated crack growth life (including small-crack corrections) in the smooth specimen from the total smooth specimen life. The second step is to apply these nucleation curves to predict the nucleation life to the same initial crack size in a feature of interest, equating the stress at the nucleation length in the feature geometry with the uniform stress in a corresponding virtual smooth specimen. The feature crack growth life is calculated from this crack size to failure, and then nucleation and growth lives are summed to obtain the total fatigue life. The approach has been demonstrated successfully for Ti-6Al-4V using available data from smooth specimen and fatigue crack growth tests to predict total fatigue lives in double-edge notched fatigue specimens at three different stress ratios.

## ACKNOWLEDGEMENTS

This work was performed with funding from the U.S. Air Force Research Laboratory and Southwest Research Institute under AFRL Dual Use Agreement No. F33615-03-2-5203. The technical monitor at AFRL was Patrick Golden. The support and encouragement of Jim Larsen and Jay Jira at AFRL is gratefully acknowledged.

## REFERENCES

1. Socie, D.F., Morrow, J., and Chen, W.-C. (1979). *Engng Fract. Mech.* 11, 851-860.
2. Dowling, N.E. (1979). In: *Fracture Mechanics, ASTM STP 677*, 247-273.
3. El Haddad, M.H., Smith, K. N. and Topper, T. H. (1979). *J. Engng Mater. Technol.* 101, 42-46.
4. Kitagawa, H. and Takahashi, S. (1976). In: *Proc. 2<sup>nd</sup> Int. Conf. Mechanical Behavior of Materials*, 627-631.
5. Chan, K. S. (1999). In: *Proc. 2<sup>nd</sup> Int. Symp. Gamma Titanium Aluminides*, TMS, 517-525.
6. Tanaka, K., Nakai, Y., and Yamashita, M. (1981). *Int. J. Fracture* 17, 519-533.
7. Brown C. W. and Taylor, D. (1984). In: *Fatigue Crack Growth Threshold Concepts*, TMS-AIME, 433-445.
8. “Improved High Cycle Fatigue Life Prediction,” USAF Contract F33615-96-C-5269.
9. Slavik, D.C., Duniy, T., Griffiths, J. and Kurath, P. (2000). In: *Proc. 5<sup>th</sup> National Turbine Engine High Cycle Fatigue Conference*.
10. Boyce, B. L. and Ritchie, R. O. (2001). *Engng Fract. Mech.* 68, 129-147.
11. Sheldon J.W., Bain, K.R., and Donald, J.K. (1999). *Int. J. Fatigue* 21, 733-741.
12. Lenets, Y.N., Bellows, R.S., and Merrick, H.F. (2000). In: *Proc. 5<sup>th</sup> National Turbine Engine High Cycle Fatigue Conference*.